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PG-268

IV Semester M.Sc. Examination, July - 2019
(CBCS - Y2K17/Y2K14)

MATHEMATICS

M401T : Measure and Integration

Time : 3 Hours | Max. Marks : 70

Instructions : (i) *Answer* ***any five*** *full questions.*
(ii) ***All*** *full questions carry* ***equal*** *marks.*

1. (a) Define outer measure of a set. Show that outer measure of an interval is equal to its length. **8+6**
   (b) Show that cantor's ternary set constructed from [0, 1] has measure zero.

2. (a) If E_1 and E_2 are any measurable sets then prove that $m^*(E_1 \cup E_2) + m^*(E_1 \cap E_2) = m^*E_1 + m^*E_2$ **4+5+5**
   (b) Prove that the collection of all measurable sets is an σ - Algebra.
   (c) If E is a measurable set then show that $E + y$, for any $y \in \mathbf{R}$ is measurable.

3. (a) If f and g are measurable functions on a measurable set E then prove that $f+g$, fg, max $\{f, g\}$ and min $\{f, g\}$ are measurable. **8+6**
   (b) Let E be a measurable set with $ME < \infty$ and let $\{f_n\}$ be a sequence of measurable functions defined on E. If $f : E \to \mathbf{R}$ is a measurable function, such that $f_n(x) \to f(x)$ for each $x \in E$ then prove that for $\epsilon > 0$ and $\delta > 0$ there is a measurable subset $A \subset E$ with $MA < \delta$ and an integer N such that $|f_n(x) - f(x)| < \epsilon$, $\forall n \geq N$, $x \in E - A$.

4. (a) State and prove Fatou's Lemma. Deduce monotone convergence theorem. **8+6**
   (b) Let f be a non-negative measurable function which is integrable over a measurable set E. Then show that for a given $\epsilon > 0$ there exists a $\delta > 0$ such that for every measurable set $A \subset E$ with $mA < \delta$ we have $\int_A f < \epsilon$.

P.T.O.

5. (a) State and prove Generalized Lebesgue dominated Convergence theorem. 6+8

(b) If f is a monotonically increasing function over [a, b] then prove that f' exists almost everywhere on [a, b]. Further show that

$$\int_a^b f'(x).dx \leq f(b) - f(a).$$

6. (a) Let f : [a, b] → **R** be a bounded function. Show that f is a function of 7+7 bounded variation on [a, b] in the following cases :

(i) f is monotonic (ii) f' exists

(b) Show that a function f : [a, b] → **R** is a function of bounded variation on [a, b] if and only if f can be written as a difference of two monotonically increasing functions.

7. (a) Let f be an integrable function defined over [a, b]. If $\int_a^x f(t).dt = 0$ for all 7+7 $x \in$ [a, b], then prove that $f=0$ almost everywhere on [a, b].

(b) Show that for an integrable function f on [a, b], the function

$$F(x) = \int_a^x f(t).dt + F(a),$$ we have, $F'(x) = f(x)$ almost everywhere on [a, b].

8. (a) Show that if f is absolutely continuous on [a, b] then f is integrable on 5+ [a, b] and $\int_a^x f'(t).dt = f(x) - f(a)$, $\forall x \in$ [a, b].

(b) Prove that indefinite integral of an integrable function on [a, b] is absolutely continuous on [a, b]. Further prove that if F is an absolutely continuous function on [a, b], then F is an indefinite integral of some integrable function.

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